# Lattice QCD review of charmonium

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# Introduction

Proper treatment of near-threshold states in lattice QCD is important since most exotic states lie near open charm threshold.

1. $X(3872)$ near $D^0 D^{*0}$ threshold;
2. $Z_c^+(3900)$ was discovered in $J/\psi\pi^+$ inv. mass by BESII and confirmed by Belle and CLEOc, it has quark structure $\bar{c}c\bar{d}u$ and also lies near $DD^*$ threshold;
3. $Z^+(4430)$ and $Z_c^+(4020)$, $Z_c^+(4025)$ have $\bar{c}c\bar{d}u$ quark structure and lie near $D^*D_1$ and $D^*D^*$ thresholds respectively;
4. the first lattice studies were done for $X(3872)$ and $Z_c^+(3900)$.

# Technical details

- **The variational method** The energy levels $E_n$ are extracted from the decay of two-point Green functions in Euclidean time,

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(t) \rangle = \sum_{n \geq 1} v_i^n v_j^{n*} e^{-E_n t}, \quad v_i^n = \langle 0 | \hat{O}_i | n \rangle$$

Interpolating operator $\hat{O}_i^\dagger$ creates states of an isospin $I$, charm number, a given momentum and $J^{P(C)}$ quantum numbers.
Need to solve the symmetrized eigenvalue problem

$$C^{-1}(t_0) C(t) u^n(t, t_0) = \lambda^n(t, t_0) u^n(t, t_0)$$

$$\lambda^n(t, t_0) \propto e^{-(t-t_0)E_n} [1 + \mathcal{O}(e^{-(t-t_0)\triangle E_n})]$$

B. Blossier, R.Sommer et al. [ALPHA Collab.], arXiv: 0902.1265 [hep-lat]
The effective energy levels or $\vec{p} = 0$ masses are obtained from the eigenvalues

$$m^{t_0}_{n,eff}(t + a/2) = a^{-1} \frac{\lambda^n(t, t_0)}{\lambda^n(t + a, t_0)}$$

**States well below open charm threshold**

$$m = \sqrt{E^2 - P^2}$$

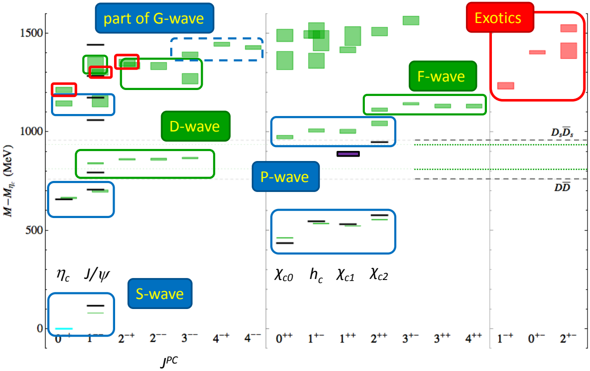
If $P = 0$ then $m = E$.
Perform extrapolations: $L \to \infty$, $a \to 0$, $m_q \to m_{phys}$
(lattice collab.: HPQCD, FNAL/MILC, $\chi$QCD).

**"Single-meson" treatment of excited states**

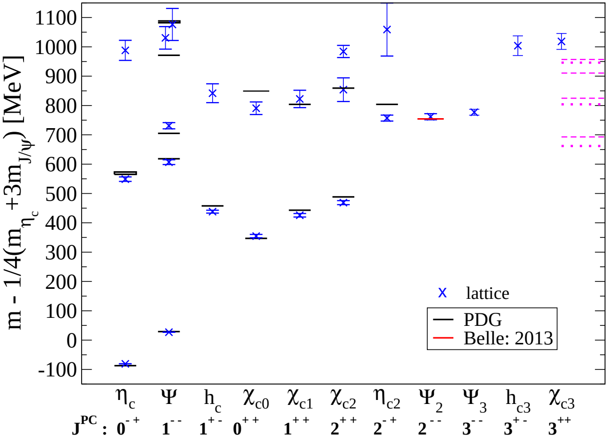
1. using only quark-antiquark interpolating fields $O \sim \bar{q}q$ for mesons;
2. assuming that all energy levels correspond to $one - particle$ states;
3. that the mass of the state equals the measured energy level $m = E$.

***These assumptions are too strong for the resonances, which are not asymptotic states. This approach also ignores the effect of the threshold and near-threshold states.***

# $\bar{c}c$ spectrum extracted by the Hadron Spectrum Collaboration




$N_f = 2 + 1$ dynamical quarks (degen. $u$ and $d$ + $s$ quark), anisotropic lattices $16^3 \times 128$ and $24^3 \times 128$ with $m_\pi \approx 400$ MeV, $L \simeq 1.9$ fm and 2.9 fm, $a_s \sim 0.12$ fm, $a_s/a_t \sim 3.5$.(Corr. with exp.: $\psi(2S)$, $\psi(3770)$, $\psi(4160)$, $Y(4260)$.)




D. Mohler, S. Prelovsek and R. Woloshin, Phys. Rev. D87, 034501 (2013)
$V = 16^3 \times 32$, $a \simeq 0.124$ fm, $m_\pi \approx 266$ MeV.

# Numerical setup

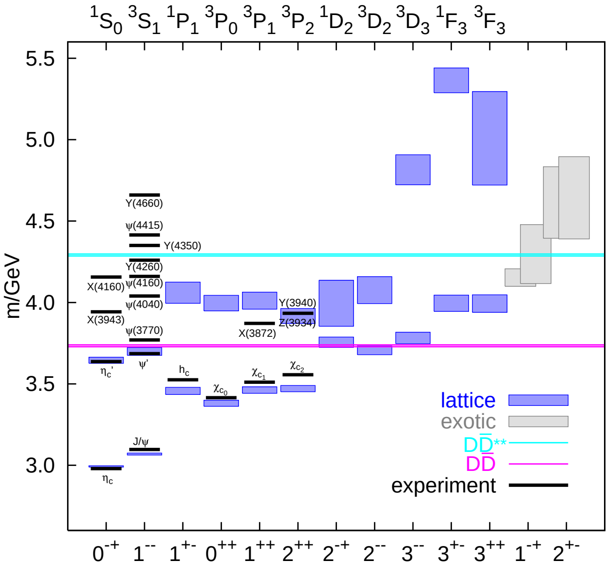
**Action**

Charm and light valence quarks and $n_F = 2$ sea quarks, the nonperturbatively improved Sheikholeslami-Wohlert (clover) Fermion action and Wilson gauge action (provided QCDSF collab.)
The charm $m_c$ is not sufficiently heavy, so the same action (relativistic) for both the charm quark and for the light sea/valence quarks is used.

**Simulation parameters**

| ID | $\beta$ | $\kappa$ | volume | $m_{\mathrm{PS}}/\mathrm{GeV}$ | $a/\mathrm{fm}$ | $L/\mathrm{fm}$ | $\kappa_{\mathrm{charm}}$ | $N_{\mathrm{conf}}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 5.20 | 0.13420 | $16^3 \times 32$ | 1.007(2) | 0.1145 | 1.83 | 0.1163 | 100 |
| 2 | 5.29 | 0.13620 | $24^3 \times 48$ | 0.400(1) | 0.0770 | 1.84 | 0.1245 | 130 |
| 3 | 5.29 | 0.13632 | $24^3 \times 48$ | 0.280(1) | 0.0767 | 1.84 | 0.1244 | 100 |

The only free parameter is the $m_c$, the authors set it by tuning $m_{1\bar{S}} = \frac{1}{4}(m_{\eta_c} + 3m_{J/\Psi})$ to exp.value $3067.8 \pm 0.4$ MeV. $a$ is from $r_0/a(\beta, k)$, $k$ is the mass parameter, $\beta$ - lattice coupling, $r_0 \approx 0.457\ fm$ - Zommer parameter.


1S0 3S1 1P1 3P0 3P1 3P2 1D2 3D2 3D3 1F3 3F3
m/GeV
5.5
5.0
4.5
4.0
3.5
3.0
Y(4660)
ψ(4415)
Y(4350)
Y(4260)
X(4160)
ψ(4160)
ψ(4040)
X(3943)
ψ(3770)
Y(3940)
Z(3934)
X(3872)
ηc'
ψ'
hc
χc0
χc1
χc2
J/ψ
ηc
lattice
exotic
DD̄**
DD̄
experiment
0-+ 1-- 1+- 0++ 1++ 2++ 2-+ 2-- 3-- 3+- 3++ 1-+ 2+-

# Rigorous treatment of near-threshold state X(3872)

$$X(3872 \to J/\psi\omega, \quad X(3872) \to J/\psi\rho$$

Consider also discrete scattering levels $DD^*$ and $J/\psi V$, where $V = \omega$ for $I = 0$ and $V = \omega$ for $I = 1$.
The eigenstates are also the s-wave scaterring states $D(\vec{p})D^*(-\vec{p})$ and $J/\psi(\vec{p})V(-\vec{p})$ with discrete momenta $\vec{p}$.
**In the absence of interaction** $p = p^{n,i} = 2\pi|n|/L$ and the scattering levels appear at

$$E^{n,i} = E_1(p^{n,i}) + E_2(p^{n,i})$$

**In the presence of interaction** the scattering levels $E = E_1(p) + E_2(p)$ are shifted with respect to $E^{n,i}$ since momentum outside the interaction region is defferent from $p^{n,i}$.
***This energy shift provides rigorous information on the*** $DD^*$ ***interaction.***
Bound states and resonances lead to levels in addition to the scattering levels.

# $X(3872)$ from $DD^*$ scattering on the lattice

The interpolatinf fields $O_i$ have to couple well to $\bar{c}c$ as well as the scattering states to study the system with $J^{PC} = 1^{++}$, $\vec{p} = 0$ and $I = 0$ or $I = 1$.

$$O^{\bar{c}c} = \bar{c}\hat{M}_i c(0) \;\; (only \;\; for \;\; I = 0)$$

$$O_1^{DD^*} = [\bar{c}\gamma_5 u(0)\bar{u}\gamma_i c(0) - \bar{c}\gamma_i u(0)\bar{u}\gamma_5 c(0)] + f_I\{u \to d\}$$

$$O_2^{DD^*} = [\bar{c}\gamma_5\gamma_t u(0)\bar{u}\gamma_i\gamma_t c(0) - \bar{c}\gamma_i\gamma_t u(0)\bar{u}\gamma_5\gamma_t c(0)] + f_I\{u \to d\}$$

$$O_3^{DD^*} = \sum_{e_k = \pm e_{x,y,z}} [\bar{c}\gamma_5 u(e_k)\bar{u}\gamma_i c(-e_k) - \bar{c}\gamma_i u(e_k)\bar{u}\gamma_5 c(-e_k)] + f_I\{u \to d\}$$

$$O_1^{J/\psi V} = \epsilon_{ijk}\bar{c}\gamma_j c(0)[\bar{u}\gamma_k u(0) + f_I\bar{d}\gamma_k d(0)]$$

$$O_1^{J/\psi V} = \epsilon_{ijk}\bar{c}\gamma_j\gamma_t c(0)[\bar{u}\gamma_k\gamma_t u(0) + f_I\bar{d}\gamma_k\gamma_t d(0)]$$

where $f_I = 1$ and $V = \omega$ for $I = 0$, while $f_I = -1$ and $V = \rho$ for $I = 1$. Polarization $i = x$ is used.

$$\bar{q}\gamma_i q$$

$$\bar{q}\gamma_t\gamma_i q$$

$$\bar{q}\overrightarrow{\nabla}_i q$$

$$\bar{q}\epsilon_{ijk}\gamma_j\gamma_5\overrightarrow{\nabla}_i q$$

$$\bar{q}\overleftarrow{\nabla}_i\gamma_i\overrightarrow{\nabla}_i q$$

$$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_i\overrightarrow{\nabla}_i q$$

$$\bar{q}\overleftarrow{\triangle}_i\gamma_i\overrightarrow{\triangle}_i q$$

$$\bar{q}\overleftarrow{\triangle}_i\gamma_t\gamma_i\overrightarrow{\triangle}_i q$$

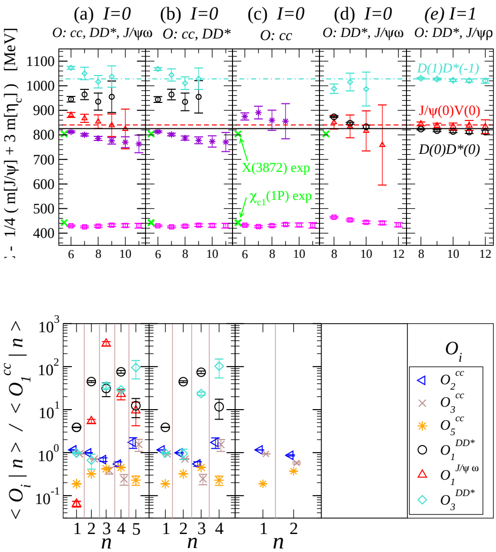
## $X(3872)$ from $DD^*$ scattering on the lattice

$$C^{-1}(t_0)C(t)u^n(t,t_0) = \lambda^n(t,t_0)u^n(t,t_0)$$

$$\frac{\langle O_i|n\rangle}{\langle O_j|n\rangle} = \frac{\sum_k C_{ik}(t)u_k^n(t)}{\sum_{k'} C_{ik'}(t)u_{k'}^n(t)}$$

Simulations: $N_f = 2$ dynamical quarks, $m_u = m_d$, $m_{val} = m_{dyn}$, $m_\pi = 266(4)$ MeV, $a = 0.1239(13)$ fm, the lattice volume $V = 16^3 \times 32$, the small spatial size $L \simeq 2$ fm is the main drawback of our simulation because $X(3872)$ is larger in size.

The present study also needs: $am_D = 0.9801(10)$, $am_D^* = 1.0629(13)$, $am_{\eta_c} = 1.47392(31)$, $am_{J/\psi} = 1.54171(43)$, $am_\rho = 0.5107(40)$ and $m_\omega = m_\rho$.


(a) I=0
O: cc, DD*, J/ψω
(b) I=0
O: cc, DD*
(c) I=0
O: cc
(d) I=0
O: DD*, J/ψω
(e) I=1
O: DD*, J/ψρ
1/4 ( m[J/ψ] + 3 m[ηc] ) [MeV]
D(1)D*(-1)
J/ψ(0)V(0)
D(0)D*(0)
X(3872) exp
χc1(1P) exp
< Oi | n > / < O1cc | n >
n
Oi
O2cc
O3cc
O5cc
O1DD*
O1J/ψ ω
O3DD*

## Results

for $I = 1$

The channel cannot contain pure $\bar{c}c$.

The lowest free levels are $D(0)D^*(0)$, $J/\psi(0)\rho(0)$ and $D(1)D^*(-1)$, their overlaps are largest with $O_{1,2}^{DD^*}$, $O_{1,2}^{J/\psi\rho}$ and $O_3^{DD^*}$

Their energies are almost equal to non-interacting energies $\Longrightarrow$ interaction in $I = 1$ is small. No extra states in addition to the scattering levels in $I = 1$.

Possible reasons: $m_u = m_d$ in these lattice simulations or not sufficient set of scattering interpolating operators.

for $I = 0$

***The first possibility:*** the stars corresponds to a weakly bound state X(3872) slightly below the $DD^*$ threshold and the circles correspond to the scattering state $D(0)D^*(0)$ which is significantly shifted up due to a large negative $DD^*$ scattering length $a_0^{DD^*} = lim_{p\to 0} tan(p)/p$

***The second possibility:*** the opposite case to the first one, attr. int. with $a_0 > 0$. It is ruled out because the analysis of lattice data leads to

$$a_0^{DD^*} = -1.7 \pm 0.4\ fm \quad \rightarrow \quad m(X(3872)) - (m_{D^0} + m_{D^{0*}}) = -11 \pm 7\ MeV$$

# Search for $Z_c^+(3900)$ exotic state

BESIII, Belle: $Z_c^{\pm}(3900)$, CLEO: $Z_c^0(3900)$,
$m(Z_c^+) = 3899.0 \pm 3.6 \pm 4.9$ MeV, suggest quarks structure $\bar{c}c\bar{d}u$.

$$Z_c^+ \to J/\psi\pi^+,$$

$J$ and $P$ quantum numbers are experimentally unknown
$I = 1$ and $C = C_{J/\psi}C_{\pi^0} = -1$
The search on the lattice by simulating the channel with $J^{PC} = 1^{+-}$ and $I = 1$ was performed with $N_f = 2$ of dynamical quarks with $m_u = m_d$.
Parameters of simulations: $V = 16^3 \times 32$, $a = 0.1239(13)$ fm, $L = 1.98$ fm, $m_\pi = 266(4)$ MeV.
No candidate was found. Possible reasons: $Z_c^+(3900)$ has another quantum numbers or the set of interpolation operators are not diverse enough.

# Search for $Z_c^+(3900)$ exotic state

Six interpolators with $J^{PC} = 1^{+-}$, $I = 1$ and total momentum $p = 0$.
PS: $I_3 = 0$ and $Z_c^0$, $m_u = m_d$ so the same operators we have for the charged mesons.

$$O_1^{DD^*} = [\bar{c}\gamma_5 u(p=0)\bar{u}\gamma_i c(p=0) + \bar{c}\gamma_i u(p=0)\bar{u}\gamma_5 c(p=0)] - \{u \to d\}$$

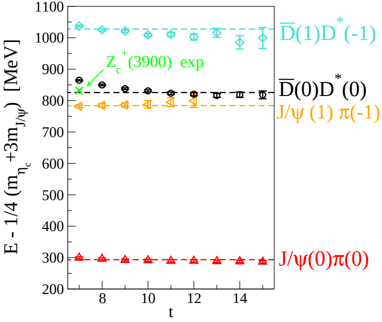
$$O_2^{DD^*} = [\bar{c}\gamma_5\gamma_t u(p=0)\bar{u}\gamma_i\gamma_t c(p=0) + \bar{c}\gamma_i\gamma_t u(p=0)\bar{u}\gamma_5\gamma_t c(p=0)] - \{u \to d\}$$

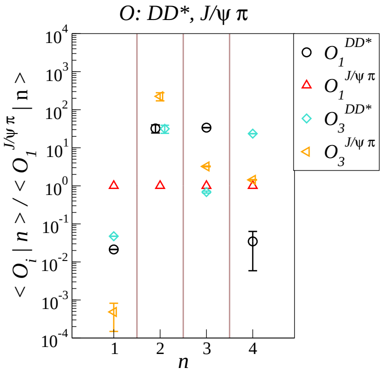
$$O_3^{DD^*} = \sum_{p=\pm 2\pi/L, e_{x,y,z}} [\bar{c}\gamma_5 u(p)\bar{u}\gamma_i c(-p) + \bar{c}\gamma_i u(p)\bar{u}\gamma_5 c(-p)] - \{u \to d\}$$

$$O_1^{J/\psi\pi} = \bar{c}\gamma_i c(p=0)[\bar{u}\gamma_5 u(p=0) - \{u \to d\}]$$

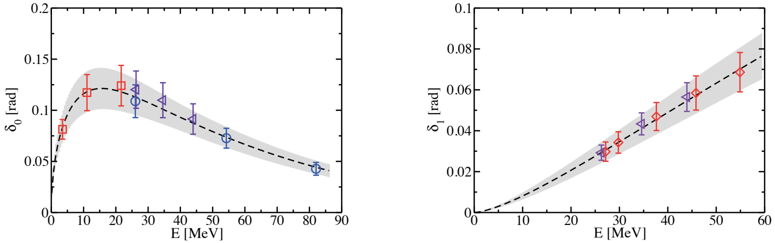
$$O_2^{J/\psi\pi} = \bar{c}\gamma_i\gamma_t c(p=0)[\bar{u}\gamma_5\gamma_t u(p=0) - \{u \to d\}]$$

$$O_3^{J/\psi\pi} = \sum_{p=\pm 2\pi/L, e_{x,y,z}} \bar{c}\gamma_i c(p)[\bar{u}\gamma_5 u(-p) - \{u \to d\}]$$


1100
1000
900
800
700
600
500
400
300
200
8
10
12
14
t
E - 1/4 (m_ηc +3m_J/ψ) [MeV]
D̄(1)D*(-1)
Z_c^+(3900) exp
D̄(0)D*(0)
J/ψ (1) π(-1)
J/ψ(0)π(0)


O: DD*, J/ψ π
< O_i | n > / < O_1^{J/ψ π} | n >
n
1
2
3
4
O_1^{DD*}
O_1^{J/ψ π}
O_3^{DD*}
O_3^{J/ψ π}

# $X(4140)$




The phase shift for s-wave and p-wave scattering of $J\psi\phi$. No resonant structure was found, ignoring of $\bar{s}s$ annihilation contribution.
For the one near-threshold bound state the phase shift have to start at $\delta(p=0)=\pi$ and fall down to $\delta(p\to\infty)=0$.

# Conclusions

1. The single meson treatment of states are not suited for excited states and resonances which are not asymptotic states of lattice correlators
2. $X(3872)$ is found below the $DD^*$ threshold $\delta m = -11 \pm 7$ MeV
3. No candidate for $Z_c^+(3900)$ in $J^{+-}$ and $I = 1$ channel
4. No $X(4140)$ resonant structure